Physics 200 Assignment 8

1. 8-24

a) stable equilibrium points: $r = 1.5$ mm, 3.25 mm
unstable equilibrium: $r = 2.4$ mm
neutral equilibrium: $r \to \infty$

b) particle bound (confined to $r < \infty$) if $E < 1$ J and $E > -5.5$ J (for $K \geq 0$)

c) for $E = -3$ J,
turning points at $r = 0.5$ mm, 3.5 mm $\to$ $0.5 < r < 3.5$ mm

d) $K = E - U \Rightarrow K_{max}$ where U most negative (at $r \approx 1.5$ mm)
$\to K_{max} = -3 - (-5.5)\,J = 2.5$ J

e) K_{max} at $r = 1.5$ mm

f) particle would need $E \geq 1$ J to reach $r \to \infty$
so $BE = 1\,J - (-3\,J) = 4$ J

2. 8-53 spring constant of pogo stick $k = 2.50 \times 10^4$ N/m
mass of child + pogo stick $m = 25.0$ kg

a) take $x = 0$ to be zero gravitational, spring potential energy
at A, $x = -0.100$ m, $v = 0$; system = child + pogo stick + Earth

$$E = \tfrac{1}{2}mv^2 + mgy + \tfrac{1}{2}kx^2$$
$$= 0 + (25\text{ kg})(9.8\text{ m/s}^2)(-0.1\text{ m}) + \tfrac{1}{2}\left(2.5\times10^4\ \tfrac{N}{m}\right)(-0.1\text{ m})^2$$
$$= +100.5\text{ J} = 100.\text{ J}$$

b) at C, $v = 0$ and $\tfrac{1}{2}kx^2 = 0$; $y = x_c$

so $E = mgx_c = 100.5$ J $\to$ $x_c = 0.410$ m

c) at B, $\tfrac{1}{2}kx^2 = 0 + mgy = 0$

so $E = \tfrac{1}{2}mv^2 = 100.5$ J $\to$ $v = 2.84$ m/s

d) K is maximum where $mgx + \tfrac{1}{2}kx^2$ is smallest

one way: $\frac{d}{dx}\left(mgx + \tfrac{1}{2}kx^2\right) = mg + kx = 0 \to x = -\frac{mg}{k} = -0.00980$ m

↑kx another way: graph $mgx + \tfrac{1}{2}kx^2$ vs x

↓mg another way: a will be ⊕ as long as $|kx| > |mg| \to$ then a ⊖ when $|mg| > |kx|$
so switches from speeding up to slowing down at $|kx| = |mg| \to x = \left|\frac{mg}{k}\right|$

e) $\tfrac{1}{2}mv_{max}^2 = K_{max}$ and $K_{max} = E - (mgx + \tfrac{1}{2}kx^2)$

$$= 100.5 - (25)(9.8)(-0.0098) - \tfrac{1}{2}(2.5\times10^4)(-0.0098)^2$$
$$= 101.7\text{ J}$$

$$v_{max} = \sqrt{\frac{2K_{max}}{m}} = 2.85\text{ m/s}$$