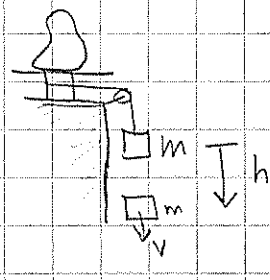
Physics 200B, Assignment 10

1. Ch 10 problem 49



conservation of mechanical energy:
initially, all in $U_i = mgh$
after m drops a distance h, it's in kinetic energy of moving m and rotating apparatus:
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

but since string wound around spool of radius r, $\omega = v/r$

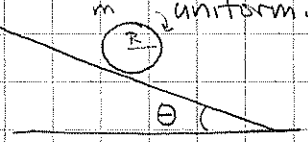
$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$

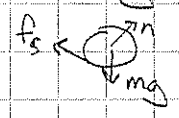
now solve for I:

$$I = \frac{2\left(mgh - \frac{1}{2}mv^2\right)}{v^2/r^2} = \frac{2mghr^2 - mv^2r^2}{v^2} = mr^2\left(\frac{2gh}{v^2} - 1\right) \quad \text{Q.E.D.}$$

2. Ch 10 problem 55

a) uniform solid disk, rolling without slipping





$\Sigma \vec{F} = m\vec{a}$:

x: $mg\sin\theta - f_s = ma_{cm}$ ①

y: $n - mg\cos\theta = 0$

also, $\Sigma\tau = I\alpha$:

$f_s R = I\alpha$

and $\alpha = a_{cm}/R \Rightarrow f_s R = I a_{cm}/R$

so $f_s = I a_{cm}/R^2$ ②

substitute ② into ①: $mg\sin\theta - \frac{I a_{cm}}{R^2} = ma_{cm}$

for uniform solid disk, $I = \frac{1}{2}mR^2$

so $mg\sin\theta - \frac{1}{2}\frac{ma_{cm}R^2}{R^2} = ma_{cm}$

$\rightarrow mg\sin\theta = \frac{3}{2}ma_{cm} \Rightarrow a_{cm} = \frac{2}{3}g\sin\theta$

compare to hoop: $I = MR^2 \Rightarrow mg\sin\theta - \frac{mR^2a_{cm}}{R^2} = ma_{cm}$

$\rightarrow mg\sin\theta = 2ma_{cm} \Rightarrow a_{cm} = \frac{1}{2}g\sin\theta$

b) $f_s = I a_{cm}/R^2 = \frac{1}{2}mR^2 a_{cm}/R^2 = \frac{1}{2}ma_{cm} = \frac{1}{3}mg\sin\theta$

and $n = mg\cos\theta$ so coeff. of friction $\mu_s \geq \frac{f_s}{n} = \frac{\frac{1}{3}mg\sin\theta}{mg\cos\theta} = \frac{1}{3}\tan\theta$