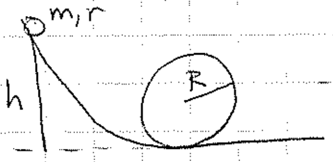
3. Ch 10 problem 79



a) to complete loop, at top n just barely > 0

$n \to 0$ so

$mg = mv^2/R \Rightarrow v^2 = gR$

using conservation of mechanical energy,

$$\underbrace{mgh}_{\text{at } h} = \underbrace{\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mg2R}_{\text{at top}}$$

and for a solid sphere, $I = \frac{2}{5}mr^2$ (m, r of sphere)

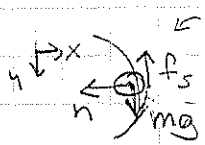
$\rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\omega^2 + mg2R$ and rolling: $v = r\omega$

$\rightarrow gh = \frac{1}{2}v^2 + \frac{1}{5}v^2 + 2gR \Rightarrow gh = \frac{7}{10}v^2 + 2gR$ but $v^2 = gR$

$= \frac{7}{10}(gR) + 2gR = 2.7gR$

$\Rightarrow h = 2.7R$

b) at P, if $h = 3R$,
$\hookrightarrow y = R$



$\Sigma F_c = n = ma_c = mv_p^2/R$

$\Sigma F_y = mg - f_s = ma_t$

since $n = mv_p^2/R$, we need v_p^2

conservation of energy: $mg(3R) = \frac{1}{2}mv_p^2 + \frac{1}{2}I\omega_p^2 + mgR$

again, $I = \frac{2}{5}mr^2$, $\omega_p = v_p/r$

so $mg2R = \frac{1}{2}mv_p^2 + \frac{1}{2}(\frac{2}{5}mr^2)(v_p/r)^2 = \frac{1}{2}mv_p^2 + \frac{1}{5}mv_p^2$

$= \frac{7}{10}mv_p^2$

$\Rightarrow v_p^2 = \frac{10}{7}(2gR) = \frac{20gR}{7}$

$n = \frac{m}{R}\left(\frac{20gR}{7}\right) = \frac{20}{7}mg$ in $-x$ direction $\leftarrow$

now to roll without slipping, $mg - f_s = ma_t$ ① (ΣF_y)

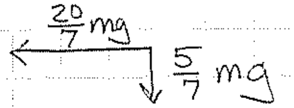
and $f_s r = I\alpha$ ② $(\Sigma \tau)$

where $I = \frac{2}{5}mr^2$, $\alpha = a_t/r$

$\Rightarrow f_s r = \frac{2}{5}mr^2 a_t/r \Rightarrow f_s = \frac{2}{5}ma_t$

so $mg - \frac{2}{5}ma_t = ma_t \Rightarrow mg = \frac{7}{5}ma_t \Rightarrow a_t = \frac{5}{7}g$

which means $\Sigma F_y = ma_t = \frac{5}{7}mg \downarrow$



force components