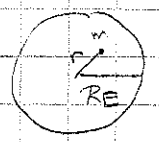
Ch 15 problem 69

a) 


gravitational force on m: $F_g = \frac{GmM}{r^2}$, opposite to r

where M = mass inside r = $\rho \frac{4}{3}\pi r^3$ since $\rho = \frac{M}{V}$

$\Rightarrow F_g = -\frac{Gm(\rho \frac{4}{3}\pi r^3)}{r^2} = -\frac{4}{3}\pi \rho G m r$

b) since $\vec{F} = -k\vec{x}$, where $k = \frac{4}{3}\pi\rho G m$, we have SHM

& $\vec{x} = \vec{r}$

period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{m}{\frac{4}{3}\pi\rho G m}} = 2\pi\sqrt{\frac{3}{4\pi\rho G}}$

time to arrive at other side $= \frac{1}{2}T = \pi\sqrt{\frac{3}{4\pi\rho G}} = \sqrt{\frac{3\pi}{4\rho G}}$

$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{5.98\times10^{24}\ kg}{\frac{4}{3}\pi(6.37\times10^6\ m)^3} = 5.523\times10^3\ kg/m^3$

so $\frac{1}{2}T = \sqrt{\frac{3\pi}{4(5.523\times10^3\ kg/m^3)(6.67\times10^{-11}\ \frac{N\cdot m^2}{kg^2})}} = 2529\ s$

$= 42\ min$

or $\frac{1}{2}T = \sqrt{\frac{3\pi}{4(M/\frac{4}{3}\pi R_E^3)G}} = \sqrt{\frac{\pi^2 R_E^3}{MG}}$