PHYSICS 200 LAB 1
Introduction to motion and to experimental uncertainty

Introduction: Experimental uncertainty, propagation of uncertainties, and comparison of measured quantities

Whenever a measurement is made, there is always some uncertainty associated with the value that results from that measurement. This uncertainty may be due to (unavoidable) imperfections in measuring instruments, in the phenomenon being measured, and/or in the observer's judgments. Our task in taking measurements is not to eliminate uncertainty (that is impossible!) but to minimize and quantify the uncertainty; in other words, the value for an uncertainty must be estimated in some way by you.

Note: the terms “error” and “uncertainty” are often used interchangeably, but we will tend to use the term uncertainty, since “error” suggests a mistake. Carelessness or bias in reading an instrument, in recording measurements, or in performing calculations are not errors or uncertainties in this sense of the word, but mistakes, and must be avoided.

Definitions:

- *Absolute uncertainty*: the uncertainty in a result, expressed in the same units as the result.
  Example: A length measurement of 4.84 cm might have an absolute uncertainty of 0.08 cm. You would write this as 4.84 ± 0.08 cm.
- *Relative uncertainty*: the fractional uncertainty in a measurement or result.
  Example: The relative uncertainty of the measurement above is 0.08/4.84 = 0.017. Since this is a ratio, it has no units.
- *Percent uncertainty*: the relative uncertainty expressed as a percentage (multiply by 100).
  Example: The percent uncertainty of this measurement is 0.017 * 100 = 1.7%.
- *Difference or discrepancy*: the difference between a result and the theoretical or expected value. This is not the same as uncertainty or error in our terminology!
- *Percent difference:* the difference between the accepted value and the measured value divided by the accepted value and then multiplied by 100. That is, $\mathrm{PD} = \left|\frac{\mathrm{Acc.Value} - \mathrm{Meas.Value}}{\mathrm{AcceptedValue}}\right| \mathrm{x}100$. The absolute value lines simply mean that this is always a positive quantity.
  Example: if the accepted value for the length we have measured is 5.00 cm, the percent difference is
  $$\left|\frac{5.00cm - 4.84cm}{5.00cm}\right| x100 = 3.2\%$$

Estimation of uncertainty:

There are two methods that may be used to obtain a measured value and its uncertainty. In the first method, a number of repeated measurements are made. The mean of these measurements may be used as the best estimate of the value itself, and the standard deviation of the measurements may be used as the best estimate of the uncertainty in the value. In the second method, the experimenter makes a single measurement and estimates an uncertainty based on his/her past experience and understanding of the measurement technique being used. Sometimes a combination of both methods is used.

Expressing a measured value with its uncertainty:

Whenever we take a measurement we usually express it numerically with its absolute uncertainty, as 4.84 ± 0.08 cm. Always round the uncertainty to 1 (sometimes 2, especially if the first digit in the uncertainty is 1) significant figures. Then round the result to the same number of decimal places as the uncertainty. For example, 12.4672938 ± 0.0034221 should be reported as 12.467 ± 0.003. Note: **Do not round intermediate results!** Keep all digits (or at least many) until the very end, then round.

Propagation of uncertainties:
Many quantities of interest in physics are calculated from several measured values. (For example, the volume of a rectangular solid is found by measuring its length, width, and thickness, and then multiplying them.) To find the uncertainty of the calculated result, the uncertainties of the measured values must be "carried through" the calculation. This process is also called "propagating" uncertainties or the propagation of error.

Our rules for the propagation of uncertainties:

1. Add absolute uncertainties when ADDING or SUBTRACTING.

2. Add relative (or percent) uncertainties when MULTIPLYING or DIVIDING.

3. For quantities that are multiplied by a constant (number with no uncertainty), multiply the absolute uncertainty by that constant. Example: if r=3.2±0.1, 2r = 6.4±0.2.

4. For quantities that are raised to a power (2, 3, 1/2, 1/3), multiply the relative uncertainty by that power. EX. $\sqrt{9\pm1\%} = 3 \pm 0.5\%$; $(5.0 \pm 2.0\%)^2 = 25 \pm 4.0\%$

The following examples will illustrate these rules and the process:

EXAMPLE #1 A roof 7.315 meters wide is to be covered by three 2.438 meter long sheets of plywood. The plywood is accurate to within ± 0.3 centimeters (cm). How long are three sheets when placed end-to-end?

Total length = (2.438 ± 0.003 m) + (2.438 ± 0.003 m) + (2.438 ± 0.003 m)
= 7.314 ± 0.009 m

Actually it is unlikely that all the sheets are long or that all are short, so a more careful analysis will give an uncertainty slightly less than the 0.9 cm shown. (This more careful analysis involves adding uncertainties in quadrature: each uncertainty is squared, all are added, and then the square root of the sum is taken. For the example, this method would give $(0.003^2+0.003^2+0.003^2)^{1/2}=0.005$.) For our purposes it will be sufficient to add all of the uncertainties together, as Rule 1 says.

EXAMPLE #2 What is the area of a 1.219 m by 2.438 m (4 by 8 ft.) sheet of plywood if each side is accurate to within 0.3 cm?

According to Rule 2, we must use relative uncertainties (%) when multiplying or dividing. Since the equation for area has the product of length and width in it, it is necessary to change our given absolute uncertainties (0.3 cm) into relative uncertainties before using this area equation.

Width = 1.219 m ± 0.3 cm = 1.219 m ± 0.003 m = 1.219 m ± 0.25%
(Change to RELATIVE uncertainty)
Length = 2.438 m ± 0.3 cm = 2.438 m ± 0.003 m = 2.438 m ± 0.12%

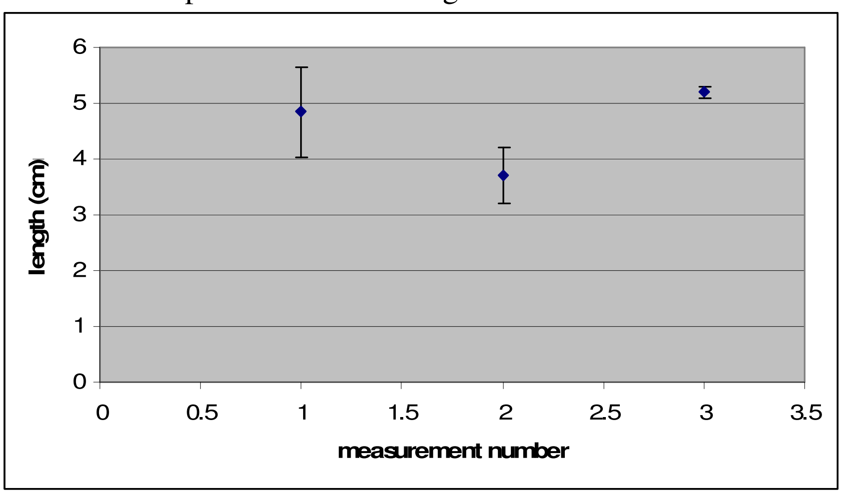
Area = Width x Length = (1.219 m ± 0.25%) x (2.438 m ± 0.12%) = 2.98 m^2 ± 0.37%
= 2.98 ± 0.01 m^2 Final answer given with ABSOLUTE uncertainty

Note that the absolute uncertainties (0.3 cm) had to be changed to meters (0.003 m) and then changed to percent uncertainties (0.25% and 0.12%) before adding them in the area equation to produce a total 0.37% "error". Finally, this percent error can be changed to an absolute value (0.01 m^2) and the result rounded to the same number of decimal places.

(As in the example above, a more careful analysis would add the relative uncertainties in quadrature: $(0.0025^2+0.0012^2)^{1/2}=0.0028=0.28\%$).

Agreement of values:

Whether two determinations of the same quantity are said to agree depends on how their difference compares to the uncertainty in the difference. If I keep measuring the same thing in the same way, about 63 percent of the time the difference between any two measurements will be within one error bar. About 90 percent of the time the measurement will be within two error bars. If the difference between two measurements is more than two error bars, there's a less than 10 percent chance that they agree: this will be our usual cutoff for saying things disagree. For example, suppose your lab group measures a length of 4.84 ± 0.08 cm and another group measures the same object to have a length of 3.7± 0.5 cm. Are these results consistent? The difference between the two values his 1.14 +/- 0.58 cm. 1.14 cm is less than twice 0.58 cm so we would say these agree (though not spectacularly well). How about 5.2 ± 0.1 cm? The difference from 4.84+/- 0.08 cm is 0.36 +/- 0.18 so they just barely agree. The difference from 3.7 +/- 0.5 is 1.5 +/- 0.6 so these do not agree. A quick check can usually be made by plotting the data points with error bars to represent the error ranges:



Note: If you are comparing a number with an uncertainty to a number that doesn't have an uncertainty, we say the numbers are in agreement if the number without an uncertainty falls within 2 times the error range of the other number.

There are two terms often associated with experimental uncertainty: accuracy and precision. The accuracy of an experiment is a measure of how close the experimental result is to the true value (that is, how small the discrepancy is). (This, of course, assumes that a true value exists and is known before the experiment is performed!) The precision of an experiment or measurement is a measure of how reproducible the result is. This is often expressed in terms of how small the uncertainty in the measurement is.

Part I: Introduction to Motion

OBJECTIVES

- To discover how to use a motion detector.
- To explore how various motions are represented on a distance (position)-time graph.
- To explore how various motions are represented on a velocity-time graph.
- To discover the relationship between position-time and velocity-time graphs.

OVERVIEW

In this lab you will examine two ways that the motion of an object moving along a line can be represented graphically. You will use a motion detector to plot distance-time (position-time) and velocity-time graphs of the motion of your own body. The study of motion and its mathematical and graphical representation is known as kinematics.

INVESTIGATION 1: DISTANCE (POSITION)-TIME GRAPHS OF YOUR MOTION

The purpose of this investigation is to learn how to relate graphs of the distance as a function of time to the motions they represent. After completing this investigation, you should be able to look at a distance-time graph and describe the motion of an object. You should also be able to look at the motion of an object and sketch a graph representing that motion.

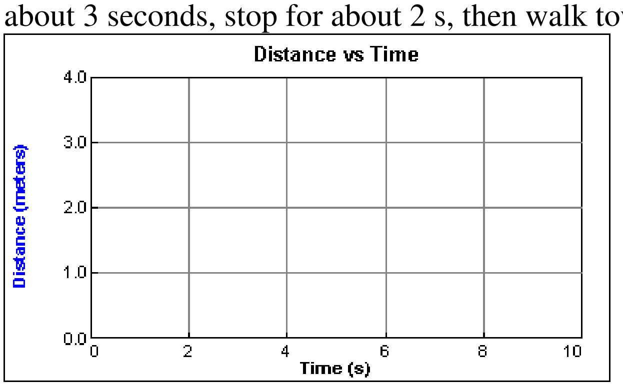
Comment: "Distance" is short for "distance from the motion detector." The motion detector is the origin from which distances are measured. The motion detector works by sending out a short pulse of high-frequency sound (ultrasound) which bounces off an object (such as you), and returns to the motion detector. From the time it takes the echo to return and the known speed of sound in air, the computer calculates the distance of the object from the motion detector. Because of the way the motion detector works, it will detect the closest object directly in front of it (including your arms if you swing them as you walk). Also, because the detector can't detect an echo that returns while the detector hasn't finished sending the pulse, it won't correctly measure anything closer than 1/2 m. *When making your graphs don't go closer than 1/2 m from the motion detector.*

Activity 1-1 : Making and Interpreting Distance-Time Graphs

1. Be sure that the interface is connected to the computer and turned on, and the motion detector is plugged into the interface. Start Logger Pro 3.1 (Desktop icon), open the RealTime Physics | Mechanics folder, and open the file called Distance [long name=> L01A1-1a (Distance).xmbl.]

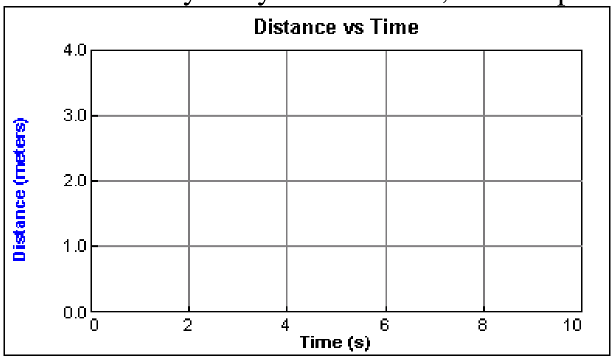
2. Make distance-time graphs for the following walking speeds and directions, sketching the resulting graphs on the axes below ("sketch" means draw the general features of the graph; don't worry about little bumps or wiggles). Click Collect to begin graphing. There's a short time delay between your motion and the graph.

a. Start about 1/2m from the detector and walk away from the detector (origin) slowly and steadily for about 3 seconds, stop for about 2 s, then walk toward the detector at the same speed.



Question 1-1: Describe the difference between the part of the graph made by walking toward and the part made by walking away from the motion detector.

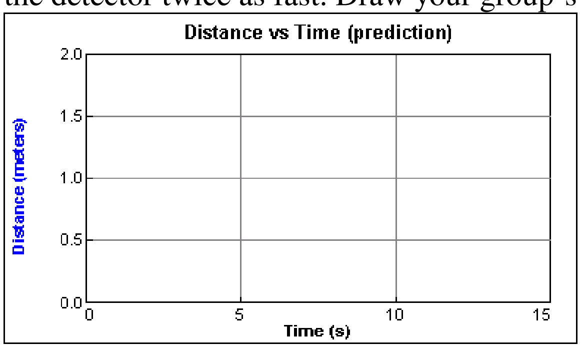
b. Walk slowly away for about 3 s, then stop for about 2 s, then walk away quickly.

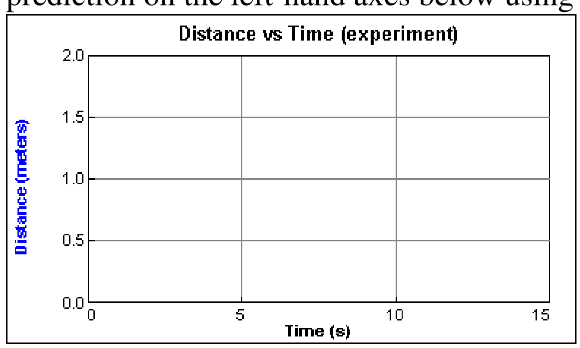


Question 1-2: Describe the difference between the part of the graph made by walking slowly and that made by walking quickly.

Comment: It is common to refer to the distance of an object from some origin as the *position* of the object. Since the motion detector is at the origin of the coordinate system, it is better to refer to the graphs you have made as *position-time* graphs rather than distance-time graphs.

Prediction 1-1: As a group, predict the position-time graph produced when a person starts l m from the detector, walks away from the detector slowly and steadily for 5 s, stops for 5 s, and then walks toward the detector twice as fast. Draw your group's prediction on the left-hand axes below using a solid line.





3. Test your prediction. Open the experiment file called **Away and Back** to set up the software to graph position over a range of 2 m for a time interval of 15 s. Move in the way described in Prediction l-l, and graph your motion. When you are satisfied with your graph, sketch the result on the right axes above. Also print out a copy of the graph.

Question 1-3: Is your prediction the same as the final result? If not, describe how you would move to make a graph that looks like your prediction.

Activity 1-2: Matching a Position-Time Graph

By now you should be pretty good at predicting the shape of a position-time graph of your movements. Can you do things the other way around by reading a position-time graph and figuring out how to move to reproduce it? In this activity you will move to match a position graph shown on the computer screen.

1. Open the experiment file called Position Match. A position graph like that shown below will appear on the screen. Delete any other data remaining from previous experiments (choose Data, then Delete Data

Set | Latest). This graph is stored in the computer so that new data can be collected without erasing the Position Match graph.

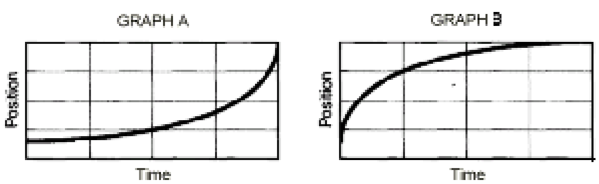
2. Move to match the Position Match graph on the computer screen. You may try a number of times. Print out a good effort.

Question 1-4: Describe your motion in words, and comment on the difference in the way you moved to produce the two differently sloped parts of the graph you just matched. Be specific.

Activity 1-3: Other Position-Time Graphs

Open the Distance Graphs file again.

1. Can you make a curved position-time graph? Try to make each of the graphs shown below.



2. Describe how you must move to produce a position-time graph with each of the shapes shown.

Graph A answer:

Graph B answer:

Question 1-5: What is the general difference between motions that result in a *straight-line* position-time graph and those that result in a *curved-line* position-time graph?

! Checkpoint 1

INVESTIGATION 2: VELOCITY-TIME GRAPHS OF MOTION

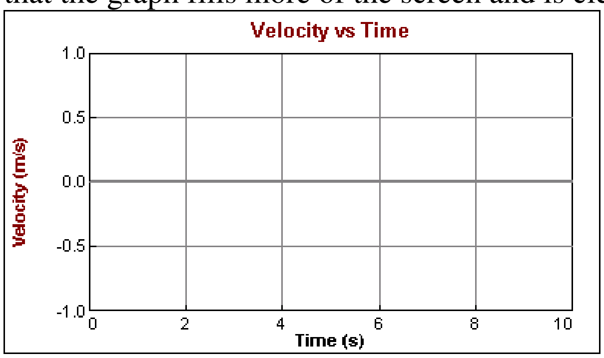
Another way to represent your motion during an interval of time is with a graph that describes how fast and in what direction you are moving. This is a *velocity-time* graph. *Velocity* is the rate of change of position with respect to time. It is a quantity that takes into account your speed (how fast you are moving) and also the direction you are moving. Thus, when you examine the motion of an object moving along a line, the direction the object is moving is indicated by the sign (positive or negative) of the velocity. A good way to learn to interpret velocity-time graphs is to create and examine graphs of your own body motions, as you will do in this investigation.

Activity 2-1: Making Velocity Graphs

1. Open the experiment file Velocity Graphs. Click on the 5 s label on the time axis and change it to 10 s.

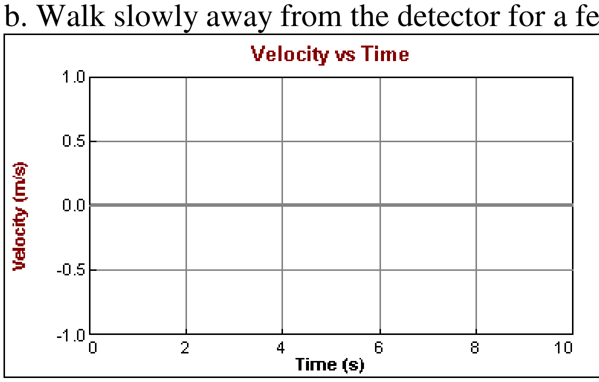
2. Graph your velocity for different walking speeds and directions as described below, and sketch your graphs on the axes. (Just draw smooth patterns; leave out smaller bumps that are mostly due to your steps.)

a. Make a velocity graph by walking away from the detector slowly and steadily for a few seconds, then immediately walk toward the detector with the same speed. You may want to adjust the velocity scale so that the graph fills more of the screen and is clearer. Then sketch your graph on the axes.



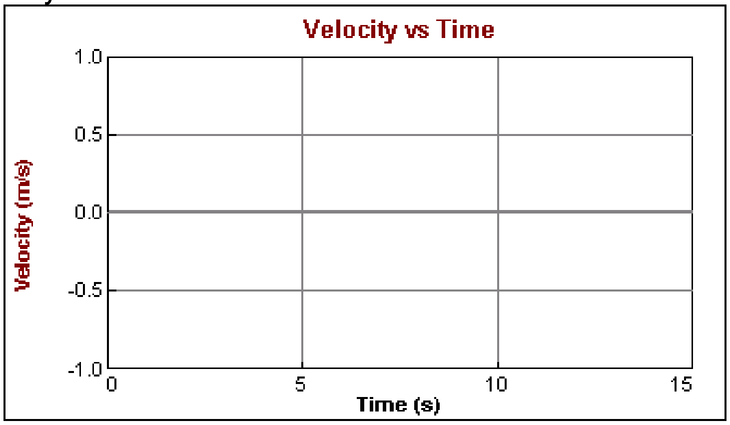
Question 2-1: How is the part of the velocity-time graph for motion away from the detector different from the part for motion *toward* the detector?

b. Walk slowly away from the detector for a few seconds, then walk away about twice as fast.



Question 2-2: What is the most important difference between the part of the graph made by slowly walking away from the detector and the part made by walking away *more quickly?*

Prediction 2-1: Draw on the axes below your *prediction* of the velocity-time graph produced if you walk away from the detector slowly and steadily for about 5 s, stand still for about 5 s, and walk toward the detector steadily about twice as fast as before.



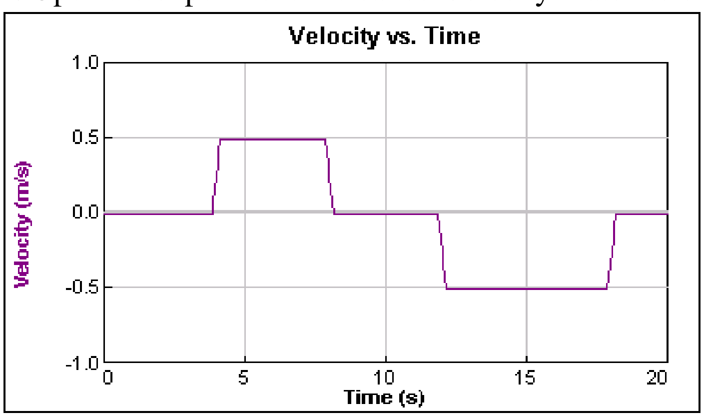
3. Test your prediction. (Be sure to adjust the time scale to 15 s.) Repeat until you think your motion matches the description well. Print out a good effort. Be sure the 5 s you spend standing still shows clearly.

Question 2.3: Comment on how well your prediction matched your graph. Were there any major differences? If so, what caused them?

Activity 2-2: Matching a Velocity Graph

In this activity, you will try to move to match a velocity-time graph. This is often much harder than matching a position graph as you did previously. In fact, some velocity graphs that can be invented cannot be matched!

1. Open the experiment file called Velocity Match to display the velocity-time graph shown below.



Prediction 2-2: Describe in words how you would move so that your velocity matches each part of this velocity-time graph.

0 to 4 s:
4 to 8 s:
8 to 12 s:
12 to 18 s:
18 to 20 s:

2. Begin graphing, and move so as to imitate this graph. You may try a number of times. Work as a team and plan your movements. Print out your group's best match.

Question 2-4: Describe how you moved to match each part of the graph. Did this agree with your predictions?

Question 2-5: Is it possible for an object to move so that it produces an absolutely vertical line on a velocity-time graph? Explain.

Question 2-6: Did you run into the motion detector on your return trip? If so, why did this happen? How did you solve the problem? Does a velocity graph tell you where to start? Explain.

! Checkpoint 2

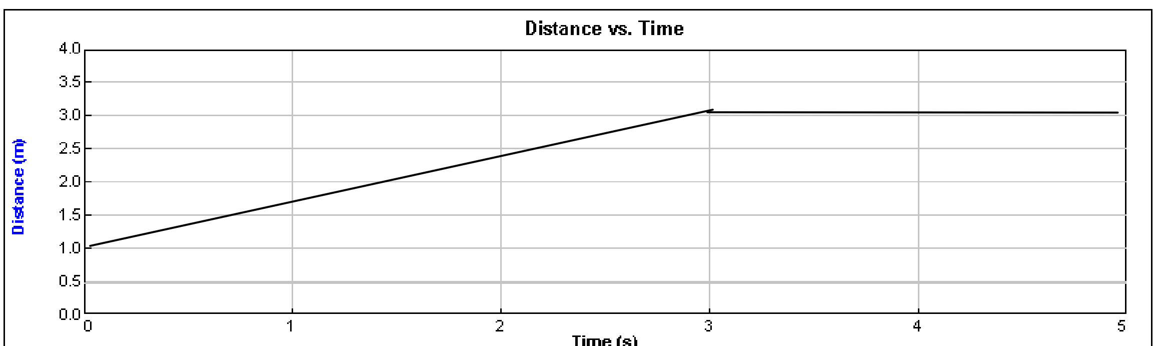
INVESTIGATION 3: RELATING POSITION AND VELOCITY GRAPHS

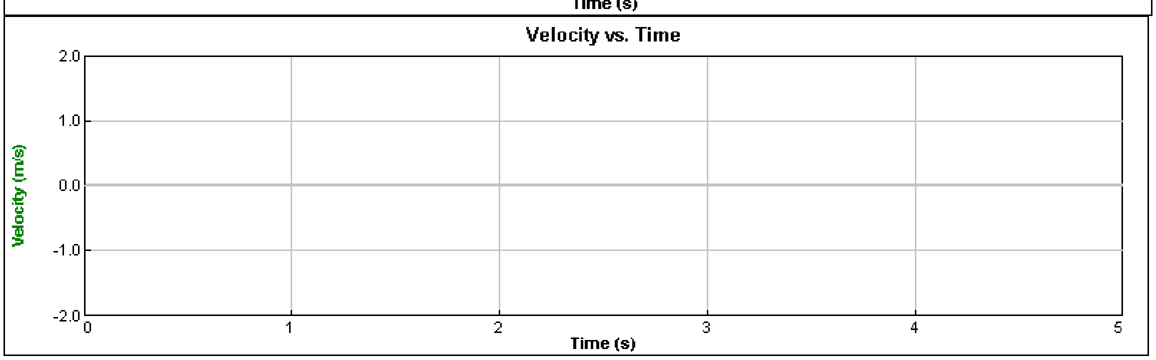
You have looked at position-time and velocity-time graphs separately. Since position-time and velocity-time graphs are different ways to represent the same motion, it is possible to figure out the velocity at which someone is moving by examining her/his position-time graph. Conversely, you can also figure out how far someone has traveled (change in position) from a velocity-time graph.

Activity 3-1: Predicting Velocity Graphs From Position Graphs

1. Open the experiment file called Velocity from Position. Clear any previous data.

Prediction 3-1: Study the position-time graph below and predict the velocity-time graph that would result from the motion. Use a dashed line to sketch your prediction of the corresponding velocity-time graph on the velocity axes.





2. Test your prediction. After each person has sketched a prediction, begin graphing, and do your group's best to make a position graph like the one shown. Walk as smoothly as possible.

When you have made a good duplicate of the position graph, use a solid line to draw the actual velocity-time graph. (Do not erase your prediction.) Also print out a copy.

Question 3-1: Discuss any differences between your prediction and the velocity-time graph you obtained.

Question 3-2: How would the position graph be different if you moved faster?

Question 3-3: How would the velocity graph be different if you moved faster?

! Checkpoint 3

Activity 3-2: Calculating Average Velocity

In this activity, you will find an average velocity from your velocity-time graph in Activity 3-1 and then from your position-time graph. You'll do this for the period of time when you were actually moving (when the speed is nonzero).

1. You can use the Logger Pro software to find the average (mean) value directly from the velocity-time graph. Click and drag to select the portion of the velocity-time graph for which you want to find the mean value (be sure to use the part of the graph that represents the time when you were moving). Next use the statistics feature (Analyze, then Statistics) to read the mean value of velocity and the standard deviation of the velocity during this portion of the motion. The standard deviation gives you a measure of the uncertainty (error) in your determination of the velocity.

Mean velocity (m/s): ____________________ Standard deviation (m/s): __________________

Comment: Average velocity during a particular time interval can also be calculated as the change in position divided by the change in time. (The change in position is often called the displacement.) As you have observed, the faster you move, the steeper your position-time graph becomes. The slope of a position-time graph is a quantitative measure of this incline. The size of this number tells you the speed, and the sign tells you the direction.

2. Use this method to calculate your average velocity from your position graph in Activity 3-l. Click in the position-time graph. Click Analyze|Examine and use the cursor to read the position and time coordinates for two typical points *while you were moving.* (For a more accurate answer, use two points as far apart as possible but still typical of the motion, and within the time interval in which you took velocity readings in part 1.)

	Position (m)	Time (s)
Point 1		
Point 2		

Calculate the change in position (displacement) between points 1 and 2; that is, position 2 – position 1. Also calculate the corresponding change in time *(time interval).* Divide the change in position by the change in time to calculate the *average* velocity. Show your calculations below.

Change in position (m)	
Time interval (s)	
Average velocity (m/s)	

Question 3-4: Is the average velocity positive or negative? Is this what you expected? Explain.

Question 3-5: Compare the average velocity you just calculated from the position graph to the average velocity you found from the velocity graph. Do they agree (within errors)? Explain. Do you expect them to agree? Why or why not?

3. For motion with a constant velocity, the slope of the position-time graph for that time period is constant (the position-time graph is a straight line). The fit routine allows you to find the equation of the line that best fits your position-time graph, and from this equation you can obtain the slope of the graph and thus the velocity. To use the fit routine, uncheck Analyze|Examine, then click on the position-time graph and select the portion of the graph that you want to fit. Next, under Analyze, select a linear fit, y = mx + b. Record the equation of the fit line below.

Equation of fit line:

Question 3-6: What is the meaning of b?

Question 3-7: Write down the value of the average velocity obtained from the slope of the graph. Does this value of velocity agree with the value from part 1? Explain. Is this what you expected? Explain.

Question 3-8: Which of these three methods do you think is best for finding the average velocity? Why?

! Checkpoint 4

Part II: Measurement & Uncertainty (or, Sink or float?)

Objectives and overview:
This experiment will give you practice in making measurements of length and mass. In addition, it will illustrate how uncertainties in your measurements are propagated through computations. Refer to the introductory material on uncertainties for the specific rules to use in your computations. The techniques practiced in this lab will be used in many of your laboratory exercises to follow.

1. VOLUME
   a. Measure the quantities necessary to determine the volume of the object provided. The uncertainty in each of your measurements and the uncertainty in the average can be estimated in several ways. Discuss how you plan to do it, and check with your instructor.

! Checkpoint 5

Record the values and proper units.

   b. Determine the volume of the object and estimate the uncertainty in the calculated value (both absolute and relative uncertainty required).

   V = ____________ ± __________ = ____________ ± __________ %

2. MASS
   a. "Weigh" your object on a scale and record its mass, the associated uncertainty, and the appropriate unit.

3. DENSITY
   Determine the density of your object. Propagate your uncertainties through this calculation.

   ρ = ____________ ± __________ = ____________ ± __________ %

4. INTERPRETATION
   Will your object float in tap water (density 1.00 g/cm^3)? Make a prediction, then try it. Discuss the results with your instructor.

! Checkpoint 6